Machine Learning Based MIMO Equalizer for High Frequency (HF) Communications

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Abstract-Utilization of multiple-input multiple-output (MIMO) systems as a means of increasing channel capacity has been an area of increasing consideration in radio communications. However, less study has been devoted to MIMO in the high-frequency band. This research is important because high-frequency communication using MIMO allows for international communication at long distances using lower power consumption than many other approaches. The inter-symbol interference caused by the selective fading of multiple received signals and the randomness of the ionospheric conditions means there is a need for a novel solution. The purpose of this research is to introduce two machine learning approaches that can adaptively apply equalization algorithms to address fading and optimize equalization parameters for given ionospheric conditions. The novelty of our approach lies in two main factors. The first is that our approach allows for a software-defined radio to switch equalization algorithms depending on conditions during run-time. The second is that we optimize this selected algorithm further by using two machine-learning approaches. The first proposed cognitive engine model, which utilizes a genetic algorithm, demonstrates the validity and advantage of using a cognitive engine to select optimal equalization parameters at the receiver under the problems created by utilizing the high-frequency band. This approach acts as a comparison for our second. We then propose a second cognitive engine, the adaptive manipulator, which optimizes not only by selecting equalization parameters but also continually changes the equalization algorithm. Finally, we compare the performance of the proposed cognitive engine models with state-of-the-art algorithms.

Index Terms—HF Communications, MIMO, Reinforcement Learning, Equalization, Q-learning, Genetic Algorithm

I. INTRODUCTION

The high frequency (HF) band, ranging from 3-30 MHz, has been a focus of study for multiple decades. Transmissions are made in the HF band by bouncing signals off the ionosphere, resulting in long-range communications. Subsequently, the HF band has been a main medium for various emergency, military, and hobbyist applications. However, despite the long-range capabilities, the ionosphere is a very turbulent channel and can change based on multiple atmospheric/geographical factors including "the time of day, latitude, season, and solar conditions" [1]. This can introduce significant distortion including "fading, dispersion, Doppler shift, and multipath [MP] distortion, all of which are constantly changing." [1]

Multiple-input multiple-output (MIMO) antenna systems are of great interest within radio communications, primarily in the ultra high frequency (UHF) and higher bands, but increasingly within the HF band as well. MIMO systems employ multiple transmitters and receivers in symmetric and asymmetric configurations to achieve higher bandwidth without drastically increasing the power required [2]. The resulting energy efficiency is achieved through the spatial diversity obtained by multiple antennas at the transmitter and receiver at the expense of inter-symbol interference (ISI) [3]. Typically, the way to handle ISI in single-input single-output (SISO) systems is to run iterative equalization schemes that convolutionally interleave their modulated transmitted signals and then equalize the received signals. While iterative equalization techniques are utilized easily and quickly for SISO, applying such approaches to MIMO systems in HF becomes challenging due to the frequency selective fading in the wireless channel. Moreover, conventional channel equalization methods are inefficient in coping with the ISI in MIMO systems. While a variety of equalization methods have been developed to correct the problems with MIMO systems, many only utilize one equalization algorithm at a given time [4][5][6]. This is often insufficient to effectively combat the frequency selective and multipath fading present in HF bands. Relatively little study has been devoted to further methods of correcting the effects of HF channels in MIMO [7][8]. Therefore, a novel method of selecting equalization methodology and parameters while still performing quickly, efficiently, and accurately is required to correct the problems of using MIMO systems in the HF band.

The parallel nature of using multiple receive and transmit

antennas causes the signals to be fused together on the receiver side, while the unpredictability of bouncing signals off the ionosphere makes consistent predictions using a single equalization algorithm with homogeneous parameter sets an unreliable tool. Therefore, MIMO applications in HF would benefit from a system that does not use a singular set of parameters for a given equalization algorithm. Such a system is called a cognitive engine (CE), which has been introduced in the field of cognitive radio [4]. However, most current CEs focus on transmission without regard to equalization and parameters, A CE with access to the choice of equalization algorithm as well as its parameters would allow for decreased bit error rate (BER) under a variety of atmospheric conditions while maintaining the increased bandwidth that MIMO allows [9].

Another complex issue that equalizer algorithms in MIMO systems must deal with is the tendency of the channel impulses to introduce large searching spaces for CEs that utilize machine learning techniques. Any CE that is to improve the quality of HF MIMO implementations must be able to handle these parameters quickly and effectively. To complicate the issue further, any implementation of MIMO in HF must find the optimal values for all parameters in combination with one another rather than local optima. The reason for this is because the parameters are mathematically dependent on the equalizer selected by the CE. From this dependency, it can be concluded that the set parameter optima a CE finds lack consistency because of the differences in changing ionospheric conditions. Therefore, a CE designed for MIMO in the HF band must be able to handle parameters rapidly, must select an equalization algorithm according to current ionospheric conditions, and must have a selection of equalizers that can handle the challenges of selective fading. The main contributions of this paper are summarized as follows:

- We propose two different forms of CEs. The first uses brute force to select an equalization algorithm and then optimizes equalization parameters. In this case, after the equalization method is selected it remains the same while the program runs. This CE acts as a baseline for comparison to not using a CE. The second CE uses a new facilitating design called the Adaptive Manipulator (AM) which dynamically changes both the equalization algorithm in use as well as its parameters.
- The CEs select between implementations of least-squares (LS) based maximum likelihood (ML), zero-forcing (ZF), and minimum mean-square error (MMSE) equalizers.
- We implement a genetic algorithm based CE in MATLAB that selects an optimal equalizer and verifies the advantage of using a CE in equalizer selection by comparing its performance to that of singular LS-ZF, LS-ML, and LS-MMSE equalizers in extensive simulations.
- We implement a new design structure called the AM

which uses Q-learning in MATLAB to optimize the parameters of a selected equalizer while the ionospheric conditions change continuously and quickly over extended periods of time to examine the effectiveness of receiver-side CEs over long-term usage. The primary difference is that the AM allows the equalization parameter to be continually changed during run-time. We demonstrate its effectiveness through simulations.

II. LITERATURE REVIEW AND BACKGROUND

To understand the novelty of a MIMO system that optimizes its own equalization parameters, detailed research was conducted to understand what state-of-the-art methods have already been suggested or applied. Despite research having been conducted on MIMO setups, HF transmission and reception, CEs, and reinforcement learning methods in the communications field, much of the previous work focuses on other potential applications. In order to generalize our model for the MIMO systems with arbitrary number of antennas at the transmitter and receiver, the work of Alamouti for the design of asymmetric transmission and reception setups has been studied [10]. In addition to asymmetric setups, the model we have created will work with and improve symmetric setups. The work of Farhang-Boroujeny et al also provided reference material for the use of Monte-Carlo (MC) simulations in MIMO systems [11].

The work of Daniels and Peters also contributed to this project as reference material for the application of MIMO in the HF band. The proposed new standard MIMO for HF transmission format for 2×2 MIMO that is back-compatible with previous standards (SISO) and was designed to support diversity coding and different forms of spatial multiplexing was useful in the design of our solution [12]. In contrast to our design, the goal was to optimize transmitted symbols instead of equalization. Similarly, the implementation of link adaptation MIMO systems using feedback from receiver to transmitter created by Chae et al influenced the adaptability of our design [9]. However, current suggestions have focused on optimizing transmission parameters rather than equalization parameters. For the CE for equalization, our design was based off preliminary work of Newman et al, Mahdi et al, and Valos et al [13]-[15]. However, despite their implementations of cognitive engines and Q-learning based optimization, few focused on equalization. Moreover, none of these works directly provide an implementation of design that considers continual optimization.

III. SYSTEM MODEL

The HF band uses the ionosphere to achieve non-line-ofsight communication between the transmitter and receiver over long distances. For the purposes of simulation, any model that attempts to test tools or technology for use in the HF band must mimic the effects of the ionosphere. In order to emulate the effects of the ionosphere on transmitted signals, the recognized standard is the Watterson ionospheric channel model [13]. The Watterson model simulates the effects of transcendental distortions caused by the ionosphere by using tap-gain functions. The tap-gain functions sample complex data-points from a bi-Gaussian distribution to simulate two magnetoionic components with Rayleigh fading. The assumption of Rayleigh fading is that the randomness of atmospheric conditions is based on the radial component of the sum of two normal distributions. The result of this combination statistically models the effects of radio communication using HF.

The primary challenge of using MIMO in the HF channel is frequency selective fading and changing channel conditions due to ionospheric effects. Instead of a singular Gaussiandistributed channel coefficient as is the case with flat fading, the MIMO Watterson Channel creates a multi-path frequency selective fading channel coefficient matrix [13]. The timevarying frequency response of the model is given by,

$$H(f,t) = \sum_{i=1}^{n} e^{-j2\pi\tau_i f} G_i(t),$$
 (1)

where n is the total number of taps, i denotes the ith tap of the channel, τ_i is the path delay of the ith tap, and $G_i(t)$ is the tap-gain function. Each of the independent tap-gain function is given by,

$$G_{si}(t) = G_{sia}(t)e^{j2\pi v_{sia}t} + G_{sib}(t)e^{j2\pi v_{sib}t},$$
 (2)

where G_{sia} and G_{sib} denote two independent complex Gaussian stationary ergodic random processes, each of which has zero-mean value and independent quadrature components with equal root mean square (RMS) value and identical spectrum. Here, v_{sia} and v_{sib} are the frequency shifts of two magnetoionic components, and t denotes the time-step. Let denote the channel coefficients matrix at time i as $\mathbf{H}_i \in \mathbb{C}^{M \times N}$,

$$\mathbf{H}_{i} = \begin{pmatrix} h_{1,1}^{i} & h_{1,2}^{i} & \cdots & h_{1,N}^{i} \\ h_{2,1}^{i} & h_{2,2}^{i} & \cdots & h_{2,N}^{i} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M,1}^{i} & h_{M,2}^{i} & \cdots & h_{M,N}^{i} \end{pmatrix}$$
(3)

where M and N are the number of antennas at the transmitter and receiver, respectively. Here, $h_{j,k}^i$ denotes the channel coefficient between the receiver j and transmitter k at time i. Then, the relationship between transmitted symbols vector $\mathbf{x}_i \in \mathbb{C}^{M \times 1}$ and received symbols vector $\mathbf{y}_i \in \mathbb{C}^{N \times 1}$ at time i is

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{x}_i + \mathbf{n}_i, \tag{4}$$

where $\mathbf{n}_i \in \mathbb{C}^{N \times 1}$ is the additive white Gaussian noise (AWGN) vector at time *i*.

We assume that the receiver has the partial channel state information (CSI). In particular, pilot symbols are transmitted at certain time slots to estimate the CSI with the LS channel estimation method. In order to estimate the CSI during the data transmission, the CSI obtained at the pilot locations are interpolated using a moving window with a fixed length. The purpose of the equalization algorithm is to equalize and denoise the received signal to recover the original signal. The maximum-likelihood (ML) algorithm gives high performance at the expense of an prohibitively expensive exhaustive search, by finding the most likely set of transmitted symbols s^* from a comprehensive dictionary $S \in \mathbb{C}^m$ to have produced the observed received signal y_i , with modulation order m. The used measure of distance is the Euclidean metric, where the distance μ to the received vector y_i is given by

$$\mu(s) = \sum_{j=1}^{N} \left| y_i^j - (H_i s)^j \right|^2$$
(5)

, where $(.)^{j}$ indicates the *j*th element of a vector. The ML equalizer then takes the vector s^* as the optimal solution,

$$s^* = \operatorname*{arg\,min}_{s \in \mathcal{S}} \mu(s) \tag{6}$$

The ML equalizer always minimizes the probability of error. However, doing so many calculations is computationally expensive [14]. An alternative to the ML is the ZF equalizer in which the interference is reduced at the cost of noise enhancement. Therefore, the algorithm functions better with a higher signal-to-noise ratio (SNR) [15]. The $M \times N$ weighting matrix of ZF can be calculated by,

$$\mathbf{W}_{ZF} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H, \tag{7}$$

where $(.)^H$ denotes the Hermitian transpose of a matrix. Similar to ZF, the MMSE algorithm finds the point of least residuals by utilizing the reduction to least squared-error. It accomplishes this minimization by calculating the weighting matrix as follows,

$$\mathbf{W}_{MMSE} = (\mathbf{H}^H + \frac{1}{\sigma^2} \mathbf{I})^{-1} \mathbf{H}^H, \qquad (8)$$

where σ^2 is the estimated noise variance and transmit power has been normalized to 1. This makes MMSE more resilient to noise, meaning it will outperform ZF in cases of low SNR [15].

To optimize the selection of equalization methodology, our research uses CEs, which apply machine learning techniques to improve the performance of software-defined radios [16]. In this paper, two separate machine learning methods are applied: genetic algorithm and Q-learning.

Genetic algorithms mimic the process of natural selection in order to attain a specific outcome [17]. Genetic algorithms start with a fitness function that defines the reward. In this application, it is given as the inverse of the BER. The actions with the highest fitness scores are selected from the population and they begin to produce children by a crossover function. The crossover function splits information between two highly successful selected parent actions and creates a hybrid. These children have a low chance of undergoing mutation, where their genes are slightly altered. This process is repeated for a given number of iterations, at which point the most fit approaches are designated the best.

In Q-learning, a set containing all possible actions is paired with a set of observed outcomes from performing said actions [18]. These pairs are stored as the axis values for a datastructure and the reward for each pair is calculated using the Bellman equations,

$$Q(s,a) = \gamma \sum_{r} P_r(r|s,a) + \gamma \sum_{t} P_t(s \xrightarrow{a} t) V^*(t), \quad (9)$$

where γ is the discount value and $V^*(t)$ is the value of taking an action to state s. The results of the Bellman equation are then stored. The result of this process is a set of all possible long-term rewards for each possible action-observation pair, referred to as a Q-table. To use the Q-table, the function greedily finds the values of the table that are of the largest rewards. The model for this greedy approach is:

$$V^*(s) = \max_{a} Q^*(s, a),$$
 (10)

where $V^*(s)$ denotes the optimal policy given starting state s. The sum of immediate reward obtained after executing action a at state s, and the discounted value of following the optimal policy $V^*(s)$ can be given by Q-value as in [19]. The decision between locating new reward values (exploring) and utilizing the constructed Q-table (exploiting) is balanced by ϵ , a Bernoulli-distributed variable designating the probability of exploration.

IV. COGNITIVE ENGINE DESIGN

This paper proposes two novel implementations of CE models. The first CE model adopts the genetic algorithm to select the optimum equalization parameters which minimize the BER for the given HF MIMO channel. This first cognitive engine cannot change the equalizer after it has been selected, which means it functions with the discrete reception of symbols, but not continuous. Benefits of using CEs to improve the performance of HF MIMO communication are demonstrated by comparing the proposed model with the stateof-the-art equalizers. The second implementation establishes a new design model for a CE called the AM which both selects the optimal equalization method and optimizes equalization parameters for a given algorithm. The performance metric of both CE models is the BER. The AM can select between zero-forcing, maximum-likelihood, or MMSE as equalization algorithms. The parameters to be optimized by AM are the window length in all cases, but σ^2 can also be modified when MMSE is used. All three equalizers utilize a series of predetermined, equally spaced pilot symbols throughout the transmission. The received symbols are divided into segments of the window length, and channel estimates are made at each pilot symbol. The channel estimates are then averaged across the containing window, and are applied in the equalization of all non-pilot symbols within the window.

The noise variance σ^2 defines the estimated variance of the AWGN present in the transmission. We assume the equalizer

does not have access to the channel SNR, and it must instead be estimated, or a fixed assumption made. For the non-adaptive state-of-the-art equalizers, the window length is fixed relative to the input signal length L and σ^2 is chosen as 0.1.

A. Genetic Algorithm based Discrete CE Model

The genetic algorithm based cognitive engine uses the MC simulation technique to estimate the performance of the first proposed genetic algorithm based CE. MC is a commonly used technique for measuring the performance of a digital communication system [14]. At each iteration, the system generates an $L \times M$ matrix of independent and identically distributed (i.i.d.) random sequences. Then, the sequences are passed through modulation and fading blocks to simulate the channel before estimation and equalization blocks attempt to reduce the fading effects in conjunction with the CE. The BER obtained by each equalizer is observed by the CE. The equalization algorithm with the lowest BER is selected. The CE then selects the nearly optimum equalization parameters using a genetic algorithm. Finally, the graphing block visually illustrates the BER versus SNR for the given channel conditions. One iteration of this MC simulation is depicted in Figure 1.



Fig. 1. Cyclic System Model

Result: Optimally equalized receive signal \hat{y} Calculate optimal BERs and parameters: $BER_{ZF}, w_{ZF} = GA(LS-ZF, Rx);$ $BER_{ML}, w_{ML} = GA(LS-ML, Rx);$ $BER_{MMSE}, w_{MMSE}, \sigma^2 = GA(LS-MMSE, Rx);$ **if** BER_{ZF} best **then** | **return** $\hat{y} = LS-ZF(w_{ZF}, Rx)$; **else if** BER_{ML} best **then** | **return** $\hat{y} = LS-ML(w_{ML}, Rx)$; **else** | **return** $\hat{y} = LS-ML(w_{MSE}, \sigma^2, Rx)$; **end**

Algorithm 1: Pseudo-code of Genetic Algorithm Based CE.

The overall performance of this system model is measured in terms of BER against a range of SNRs.The genetic algorithm CE operates according to the pseudo-code given in Algorithm 1:

B. AM and Q-Learning Based Continuous CE Algorithm

The Watterson model is used in order to mirror effects of the ionosphere. To serve the purpose of continual optimization and equalizer selection in the Watterson channel, we introduce the AM, a facilitator for the the second Q-learning based cognitive engine. The AM is a facilitator for CEs that allows for full optimization of the equalization process. The equalizers provided to the AM all perform well in MIMO operative conditions as well. The AM system structure is illustrated in Figure 2.

The AM performs two tasks: in the first stage the equalization algorithm is selected by brute force. This returns the equalizer of lowest error as well as baseline BER for future iterations of the cyclical software to use. The output of the brute force function is fed into a sentinel loop with a termination condition of a substantial growth away from the baseline BER. This means that, should the BER suddenly grow to an unacceptable level, the loop is exited and a new equalization method is selected. In this second loop, a Q-learning algorithm takes the now brute force selected equalization method and optimizes the parameters for the given equalization method. In summary, the structure is comprised of two loops: the first uses brute force to select an equalization algorithm based on lowest BER. Then the second loop repeatedly applies Q-learning to optimize the parameters of the selected equalization method to further reduce BER. In this case, the Q-learning has an action space containing all of the parameters to be optimized. The inverse of the BER is the reward for the Q-learning algorithm. The sentinel for the second loop is the baseline error calculated in the first loop. Therefore, if the BER grows too high the internal loop will break causing the first loop to run again and find a new equalization algorithm.



Fig. 2. AM system Model

By not assuming the equalizer will work in all conditions, the model is resilient to changes that occur in the ionosphere overtime. By allowing the model to control choice of equalizer and its adapting capabilities, more optimization can be achieved than assuming a single equalization algorithm. Dynamic equalization also allows the AM to select parameter sets according to which equalizer is chosen, which confers the benefit of reduced searching space for the Q-learning. The small space allows the algorithm to learn repeatedly, rapidly, and effectively. The mathematical dependence of the equalizer parameters set on the selected equalizer is also maintained.

The Q-learning CE operates according to the pseudo-code given in Algorithm 2. Here, θ is the discount rate, η is the learning rate, ψ is the exploration probability, λ is the chosen equalizer, and y is the optimized BER. Outer loop represents a continuous loop that terminates on system transmission end, and the inner loop represents a sentinel loop.

Algorithm 2: Pseudo-code implementation of Q-learning based CE.

V. RESULTS

A. Genetic Algorithm based Discrete CE Model

In our simulations, we consider 2×2 , 3×2 , and 2×3 MIMO systems, which use BPSK modulation with a sampling rate of 9.6×10^3 Hz. We used a pilot frequency of 2 for every two data symbols, and default noise variance of 0.1. Moreover, we assume that the equalizer block has no knowledge of the current SNR. The interpolation window for the channel estimation and noise variance in the system are either optimized by the CE or fixed in the state-of-the-art equalizers. A stream of random symbols is transmitted from each antenna of the MIMO system in a given time period.

BER, signalling rates, and spectral efficiency results for 2×2 , 3×2 , and 2×3 MIMO systems are shown in Figures [3]-[5], respectively. It can be easily observed that the cognitive engine always outperforms the equalizer parameter set without optimization in terms of BER, data rates (DR), and spectral efficiency (SE). Equalizer parameter sets without optimization show an unstable manner because of the behavior of channel conditions, whereas the CE performs more stably and accurately over the SNR, even in the random channel conditions.



Fig. 3. 2×2 MIMO comparison with state-of-the-art, 1000-iteration MC simulation using 200 symbols each antenna per iteration with 2dB SNR increments.



Fig. 4. 3×2 MIMO comparison with state-of-the-art, 1000-iteration MC simulation using 50 symbols each antenna per iteration with 5dB SNR increments.

B. Q-Learning Based Continuous CE Model

A 2×2 MIMO system is considered for the AM model using Q-learning, which uses QPSK modulation. The SNR value changes in an stochastic way between 1dB to 30dB. We set the sampling rate to 9.6×10^3 Hz. 2×10^2 random symbols are transmitted each time with the default pilot frequency of 2, window length of 10, and a variance of 0.1.

In order to provide proof of concept for the AM, the histogram in Figure 6. runs for three signals. The first component in the stack of each column represents the baseline error produced by the AM between the equalizer selection by the brute force algorithm and the optimization using Q-learning.



Fig. 5. 2×3 MIMO comparison with state-of-the-art, 1000-iteration MC simulation using 200 symbols each antenna per iteration with 5dB SNR increments.

Graph of BER for With and Without Optimzation Operating in Discrete Mode



Fig. 6. Q-Learning System model run for 2 signals as discrete operations for demonstrative purposes.

The three remaining components of the stack represent the error after optimization. In order to discretize the AM's continuous operation, the system was run three times with the same Q-learning policy.

As the SNR ratio increases the error exponentially decreases in the baseline and subsequent optimized runs. For the purposes of this example, the Q-learning algorithm was provided with 500 iterations of exploration split even with exploitation. In continuous operation, there would be a varying number of columns for each increment of the x-axis, which would be infinitely long as the cycles would not be limited as they are in the histogram above. Furthermore, each bar in the histogram would have a variable number of stacks for each x-axis increment. Despite the limitations in displaying the results, the histogram in Figure 6 provides an example of a functional implementation of an AM.

VI. CONCLUSION

MIMO promises exciting improvements in wireless transmissions in the future, including in the HF band. These applications are of particular interest for long-range communications without satellites, both in defense applications as well as in civilian communications. We present a new approach to develop cognitive engines based on both genetic techniques and reinforcement learning in the HF band for receiver-side signal equalization in MIMO systems. We demonstrate the gains achievable with CE-adjusted equalization parameters in the presence of pilot data, as well as the applicability of this technique to long-term operation in the HF channel. Additionally, we propose a new system component in the form of the AM, as a means of simultaneously ensuring quality and speed in equalization and demonstrating its feasibility in longrunning HF-MIMO receiver applications. The encouraging results for CE-based equalization in counteracting the effects of the ionosphere on HF-MIMO transmissions presented herein lend themselves to further work in applications of more complex equalization techniques, as well as more complex CE approaches. Further research could be performed using the AM structure with Q-learning utilizing experience-replay or potentially a DQN. Research could also be similarly approached for transmitter side optimization with modification.

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